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STEADY-STATE NUMERICAL SOLUTION OF VAPOR COMPRESSION REFRIGERATION UNITS

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Abstract

This article is concerned with the simulation of the steady-state operation of the four basic components of a domestic refrigeration system. Based on individual mathematical models and appropriate input parameters, equilibrium conditions are numerically searched for all components. Stable parameters are iteratively obtained after imposing that the refrigerant mass flow rate is converged, calculated in both the compressor and the capillary tube. Overall energy balance over the evaporator is also an imposed condition. Explicit equations for refrigerant properties are used. Alternative refrigerants are also considered. A C++ computer program was written and a test case simulating a typical domestic refrigerator is reported.

INTRODUCTION

The use of numerical tools in solving *real-world* engineering problems has become a common place strategy in the past decade, mainly due to the accelerated advances in microprocessor technologies and substantial improvements in *software* development. These two factors have led, ultimately, to reduction of required time for design and analysis of new industrial equipment. Many different configurations are able to be analyzed before prototype construction and testing, reducing the overall cost before a new conception finally goes to the market.

Motivated by the foregoing, this article presents numerical predictions for preliminary design and analysis of domestic refrigeration systems. The class of equipment here studied is schematically shown in Figure 1. The system consists of the four basic components, namely the compressor, evaporator, capillary tube and condenser. The physical contact between the compressor suction line and condenser liquid line is here recalled as the *heat exchanger*. This is a common device used in most domestic systems in order to ensure the necessary superheat at the compressor can inlets.

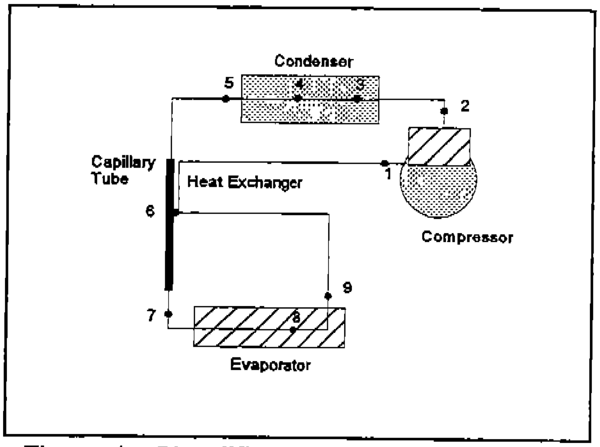

Figure 1 - Simplified scheme of a domestic refrigeration system.

Many research endeavors have been pursued in the past few years aiming the numerical simulation of such systems. These models range from sophisticated transient analyses [1,3,8,13] to more simple steady-state approaches [4,10], covering a wide range of applications such as solar-heated devices [6] and automobile air-conditioning [2]. Under this point of view, the present work would be classified as a simplified approach for performing preliminary steady-state analysis of refrigeration systems.

SYSTEM SIMULATION

The simplified cycle modeled in this work, corresponding to figure 1, is schematically shown in Fig. 2. At the compressor inlet (point 1), the refrigerant is assumed to be at a given temperature difference in relation to the evaporating temperature. This temperature difference comes from presuming a superheat at the evaporator exit (point 9) and a temperature raise due to the contact between the suction line and the capillary tube. This heat exchanger, commonly used in domestic refrigeration, ensures the necessary superheating at the compressor inlet while promoting certain subcooling before the capillary tube (point 6). For modeling purposes, no pressure drop is considered along the heat exchanger shown in figure 1.

The individual models to be seen below, together with correlations for transport and thermodynamic properties of traditional (R12) [3,7] and alternative fluids (R134a) [11,12], were solved in order to find the refrigerant states at which balance is maintained in a steady state condition. Given the ambient and internal refrigerator temperatures, in addition to all necessary data to specify the refrigeration machine, the operating conditions representing the steady-state situation can be numerically searched. To accomplish that, the computer program logic has to satisfy three balances, being the first two as follows [3]:

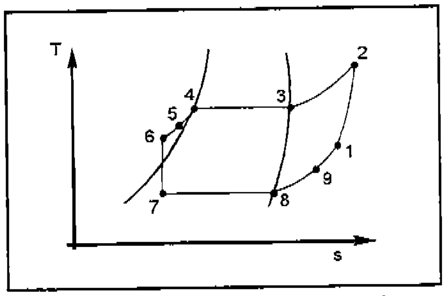

Figure 2- Diagram Txs for the analyzed cycle.

1) *The mass flow rate balance.* The refrigerant mass flow rate pumped by the compressor has to equalize that one passing through the capillary tube.

2) *The energy balance.* The overall energy balance in a refrigeration system is checked by comparing the energy gain by the refrigerant when crossing the evaporator with that of the refrigeration effect itself. A perfectly balanced system will have both quantities equal and that means a satisfied energy balance condition.

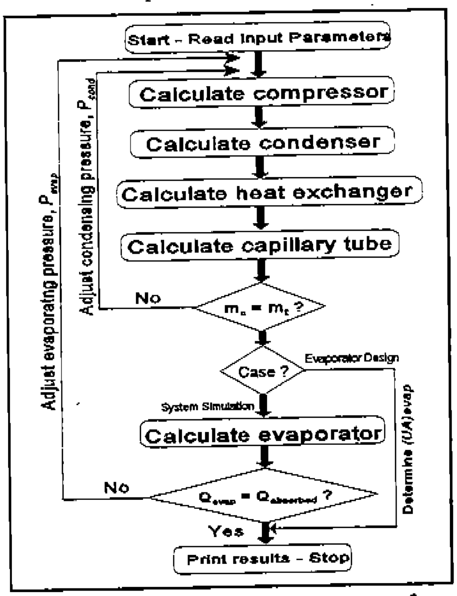

Figure 3 - Computer program for refrigeration system simulation.

The third loop mentioned in the work of Domanski & Didion (1983) [3] deals with the determination of the superheat at the evaporator exit. There, the authors determined this superheat by imposed that the mass inventory, calculated in the entire system by adding up the masses of all components, had to be equal to the input mass value. Here, for simplicity, this superheat degree at the evaporator exit is assumed to be of constant value.

The overall computer program flow chart than follows the sequence laid in Figure 3. The figure indicates two possibilities after establishing the equilibrium for the refrigerant mass flow rate. One can either stop program execution and calculate the evaporator characteristics which balance the system, or else, one can proceed and search for new evaporating pressure and temperature for a given evaporator. The first path is here recalled as ***evaporator design*** whereas the second sequence is named ***system simulation***. Below is a description of all individual mathematical models used.

COMPONENT MODELING

Compressor. Compression is assumed to follow a polytropic process of constant index n. Given the discharge and suction pressures, in addition to the compressor entrance temperature, the final discharge state at the compressor outlet can be characterized by (point 2 in figure 2):

$$v_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}} v_1 \tag{1}$$

In equation **(1)** v_1 e v_2 are the specific volumes before and after the compression and P_1, P_2 the two pressure levels which bound the compression cycle. The mass flow rate driven by the compressor can be calculated as:

$$m_c = \left(\frac{V_c}{v_1}\right)\left(\frac{w}{2\pi}\right)\eta_v \tag{2}$$

In equation (2), V_c is the compressor swept volume per revolution, w the compressor axis speed (in radians per second) and η_v the compressor volumetric efficiency. The latter is dependent upon r and C_v, two coefficients which take into consideration the compressor leakage and clearance volume, respectively [10].

Condenser. For the condenser, the overall heat rate is given by the formula,

$$Q_{cond} = (UA)_{cond}(T_{cond} - T_{amb}) \tag{3}$$

where T_{cond} and T_{amb} are the condensing and ambient temperatures, respectively. The product $(UA)_{cond}$ takes into consideration the superheated, two-phase and sub-cooled regions of the refrigerant film, conduction through the solid and the contact air-condenser metal.

Heat Exchanger. As mentioned above, the heat exchange device assures the degree of subcooling at the compressor inlet section. This modifies the original cycle as shown in Figure 2. An energy balance over the heat exchanger section can be written in terms of corresponding temperatures as $c_{pl}(T_5 - T_6) = c_{pv}(T_1 - T_9)$ where c_{pl} and c_{pv} are the specific heat for the liquid and vapor phases, respectively. In order to consider the usefulness of this contact between the two lines, an effectiveness ϵ is introduced, giving for temperature T_6,

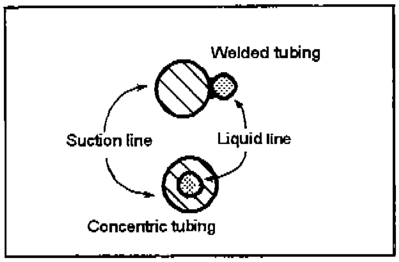

Figure 4 - Possible arrangements for heat exchanger.

$$T_6 = T_5 - \epsilon \frac{c_{pv}}{c_{pl}}(T_1 - T_9) \tag{4}$$

For $\epsilon = 1$, all heat given up by the high pressure liquid line is absorbed in the suction tube. The absence of a heat exchanger is modeled when the coefficient ϵ takes the null value. Intermediate states are simulated with ϵ less than one. Figure 4 illustrates two arrangements commonly employed by the refrigeration industry.

Evaporator. For the evaporator, an equation similar to (3) states the energy balance in that component as:

$$Q_{evap} = (UA)_{evap}(T_{evap} - T_{refr}) \tag{5}$$

where the subscripts *evap* refers to evaporating conditions whereas *refr* indicates the temperature inside the cooled compartment. The value of the product $(UA)_{evap}$, for the case of *evaporator design*, can be calculated as the one which maintain the equilibrium condition reached with the desired refrigerating and ambient temperatures.

Capillary Tube. Flow through the capillary tube is supposed to follow a Fanno line. Figure 5 illustrates the regions of different fluid phases along this device. The subcooled refrigerant entering the capillary tube suffers a sudden pressure loss due to entrance effects. Subsequently, if one considers *adiabatic* flow in the single phase region, the pressure drops linearly while temperature is kept constant. After reaching the saturation condition, the pressure gradient is increased along the flow due the fluid acceleration. Note that the inclusion of the heat exchanger device shown in figure 1 precludes the use of the adiabatic assumption over the entire capillary tube.

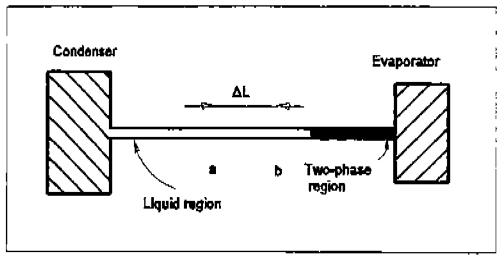

Figure 5. Refrigerant states along the capillary tube.

Balance Equations. For calculating the capillary tube, the conservation equations for *mass*, *energy* and *momentum* are used. The balance of mass between sections *a* e *b* separated by a length ΔL along the tube can be written as (see figure 5),

$$\frac{m_t}{A} = \frac{V_a}{v_a} = \frac{V_b}{v_b} = constant \tag{6}$$

where (m_t/A) is the refrigerant mass flux. Energy conservation along ΔL is given by,

$$h_a + \frac{V_a^2}{2} = h_b + \frac{V_b^2}{2} \tag{7}$$

in which the process along the capillary tube was assumed to be *adiabatic*. Momentum conservation between the same points considers both pressure and shear forces and can be further written in the form,

$$(P_a - P_b) - f\frac{\Delta L}{D}\frac{V^2}{2v} = \frac{m_t}{A}(V_b - V_a) \tag{8}$$

It is important to note that in the second term of (8) the values of V, v e f differ from section a to b. Using (6), this shear term can be rewritten as,

$$f\frac{\Delta L}{D}\frac{V^2}{2v} = f\frac{\Delta L}{D}\frac{V}{2}\left(\frac{m_t}{A}\right) \tag{9}$$

As the refrigerant flows along the two-phase region, its pressure and temperature decrease whereas the flow quality, x, increases. At a certain location, $h = h_l(1-x)+h_v\,x$ and $v = v_l(1-x)+v_v\,x$. Also, the friction factor to be used in (8) differs according to the local fluid conditions. For the two-phase region, the flow quality is determined as commented below.

Flow Quality. For a given mass flux (m_t/A), a combination of equations (6) and (7) gives,

$$h_a+\frac{V_a^2}{2} = h_b+\frac{v_b^2}{2}\left(\frac{m_t}{A}\right)^2 \tag{10}$$

Substituting local values into equation (10), applied at a general two-phase section b, one gets,

$$h_a+\frac{V_a^2}{2} = \left[h_{lb}(1-x)+h_{vb}\,x\right]+\frac{\left[v_{lb}(1-x)+v_{vb}\,x\right]^2}{2}\left(\frac{m_t}{A}\right)^2 \tag{11}$$

Assuming that all other variables are given, the flow quality x in (11) can be calculated as the only unknown.

Table 1 . Input data for refrigerator design and simulation.

Parameter	Value
Refrigerator inside temperature, T_r	46°F
Ambient temperature, T_{amb}	109.4°F
Superheat at evaporator outlet, $T_9 - T_8$	50°F
Initial superheat at compr. inlet, $T_1 - T_8$	55°F
Temperature difference at evap., T_r-T_{evap}	51°F
Capillary tube diameter, d	2.231e-3 ft
Capillary tube length, L	10.33 ft
Condenser characteristic, $(UA)_{cond}$	9.9845e-3 Btu/s.°F
Evaporator characteristic, $(UA)_{evap}$ (used only in simulation)	1.8435e-3 Btu/s.°F

Equilibrium Condition 1. The refrigerant mass flow rate pumped by the compressor is calculated with equation (2). This value is compared with the one passing through the capillary tube. Considering now an average value for the refrigerant specific volume over the entire tube, v_m, equation (8) can be integrated giving finally for the capillary tube mass flow,

$$\frac{m_t}{A} = \left(\frac{(P_7-P_6)}{v_m\left[\ln(\frac{v_7}{v_6})+2f_m\frac{L_t}{D}\right]}\right)^{\frac{1}{2}} \tag{12}$$

For estimating the mass flow rate in the capillary tube using (12), the value for the *average* friction factor f_m, for the entire tube, needs also to be prescribed. This figure can be obtained as a combination of the independent friction factors for the subcooled and two-phase regions. By adjusting the condensing pressure, equations (2) and (12) are forced to give the same mass flow rate.

Equilibrium Condition 2. The overall heat exchanged at the evaporator (equation (5)) is compared with the enthalpy gain of the refrigerant calculated with:

$$Q_{absorbed} = m_t(h_9 - h_7) \tag{13}$$

Equations (5) and (13) are compared and the evaporating pressure is varied until $Q_{absorbed}=Q_{evap}$ within some preselected tolerance.

For the *evaporator design* case, the equilibrium condition 1 above has to be satisfied. For *system simulation*, conditions 1 and 2 have to be fulfilled.

RESULTS AND DISCUSSION

This section presents preliminary numerical results based on the mathematical model just described. The equations seen above were programed using the C++ computer language and run on a personal computer. For testing calculations, a set of basic input data was used. These data correspond to typical operating conditions of domestic systems and are shown in Table 1. As mentioned before, in this model the refrigerant superheat at the evaporator exit is kept constant in all computations. The two cases here analyzed, the *evaporator design* and *system simulation*, are reported below.

Evaporator Design. In the **design** case, the evaporator conditions are kept constant while the condensing parameters are varied until the compressor drives the same mass flow as the capillary tube (see Fig. 3). During this iterative process, intermediate solutions can be monitored at the computer screen as illustrated in Figure 6. The figure indicates the adjustment of the condensing conditions while keeping the given evaporating values. Curve "C" in figure 6 represents the final converged solution, established under the convergence criterium given by $\lambda=(m_c-m_t)/m_c$ where m_c and m_t are given by equations (2) and (12), respectively.

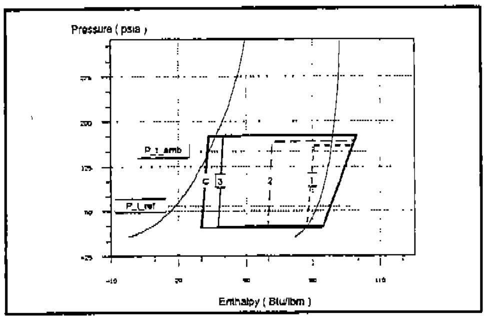

Figure 6 - Screen output for steady-state search - design case.

The influence of the tightness of the convergence parameter λ is presented in Table 2 were results for the evaporator design case are shown. The table indicates that differences up to 5.5% for COP and $(UA)_{evap}$ can be calculated if care is not taken to properly select the value of λ.

System Simulation. As mentioned before, for simulating an existing system working under the conditions given in Table 1, a secondary loop for adjusting the evaporator pressure and temperature is necessary. The coefficient λ controlling convergence takes now the form , $\lambda=(Q_{absorbed}-Q_{evap})/Q_{evap}$. Table 3 show results comparing the use or not of a heat exchanger (*equation* (4) *with different* ϵ). Also shown in the table is a comparison of calculations using traditional and alternative refrigerants. The table indicates that the use of a heat exchanger between the liquid and suction lines reduces the overall evaporating and condensing properties, while increasing the flow quality at evaporator inlet and refrigerant mass flow rate.

Further results comparing data for systems using refrigerants R12 and R134a, both with a heat exchanger installed, are also presented in the same table. Calculations indicate that the replacement of refrigerant R12 induces an increase in the compressor operating temperatures as well as in the evaporating and condensing data. At the same time, the flow quality at the evaporator inlet and the refrigerant mass flow rate are reduced. These findings might help engineers in adapting existing systems in coping with enforcement of new environmental laws.

Table **2** .Effect of convergence criterium.

Parameter	**Value**		
Refrigerant	R12		
Case	Evaporator Design (fixed superheat)		
Compressor suction temperature, T_1 (fixed)			50 °F
Evaporating temperature, T_{evap} (fixed)			-5°F
Evaporating pressure, P_{evap}			21.421 psia
Convergence criterium	λ=1.0e-3	λ=1.0e-4	Error
Condensing temperature, T_{cond}	122.05°F	120.79°F	-1,03%
Coefficient of performance, *COP*	3.6152	3.4169	-5,49%
Evaporator characteristic, $(UA)_{evap}$	0.00187 Btu/s.°F	0.00177 Btu/s.°F	-4,97%

Table **3** .Effect of refrigerant and heat exchanger on system simulation.

Parameter	**Value**		
Convergence criterium, λ	1.e-4		
Refrigerant	R12	R12	R134a
Heat exchanger efficiency, ϵ	0.0	0.95	0.95
Compressor discharge temperature, T_2	179.28°F	177.97°F	284.84°F
Evaporating pressure, P_{evap}	21.308 psia	21.416 psia	23.414 psia
Condensing pressure, P_{cond}	176.96 psia	174.11 psia	242.84 psia
Refrigerant mass flow rate, m_c	0.0017543 lbm/s	0.0018004 lbm/s	0.0011319 lbm/s

CONCLUDING REMARKS

This work showed a simplified mathematical model for simulating refrigeration systems. The simulation strategy used in order to overcome the many existing non-linearities were discussed upon. The overall calculation strategy permits both the design of evaporators working together with other components as well as simulation of existing systems. Individual mathematical models for all basic components were described. Preliminary testing results investigated the role of the convergence parameter, the use of an additional heat exchanger in the compressor suction line and the use of alternative refrigerants. The result herein stimulates further research in developing a useful and efficient tool for analyzing refrigeration systems.

REFERENCES

[1] Chi, J. , Didion, D., **A Simulation of the Transient Performance of a Heat Pump**, *International Journal of Refrigeration*, Vol. 5, no. 3, pp. 176-184 (1982).

[2] Davis, G.L., Chianese,F., Scott, T.C., **Computer simulation of automotive air conditioning, - Computers, systems and vehicle,** *Society for Automotive Engineers*, paper 720077, Automotive Engineering Congress, Detroit, 1-16, January (1972).

[3] Domanski, P., Didion, D.A., **Computer Modeling of the Vapor Compression Cycle with Constant Flow Area Expansion Device**, *NBS Building Science Series* 155, Washington D.C.,1983.

[4] Domanski, P., Didion, D.A., **A simplified cycle simulation model for the performance rating of refrigerants and refrigerant mixtures**, *Proc. 1990 ASHRAE Purdue* CFC *Conf.*, pp 466-475, July 17-20, West Lafayette, USA (1990).

[5] Downing, R.C., **Refrigerant Equations**, *ASHRAE Transactions*, vol 80, part 2, Paper 2313, 158-169, (1974).

[6] Freeman; T.L., Mitchell, J,W., Beckman, W.A., and Duffie, J.A., **Computer modeling of heat pumps and simulation of solar-heat pump systems,** *The American Society of Mechanical Engineers Winter Annual Meeting*, Paper 75-WA/Sol-3, Houston, Texas, 1-16, (December 1975).

[7] Gatecliff, F.W. and Lady, E.R., **Explicit Representation of the thermodynamic properties of refrigerants 12 and 22 in the superheat region,** *Proc. 1974 Purdue Compressor Tech. Conf.*, 187-190, (1974).

[8] Mc Arthur, J.W., **Theoretical Analysis of the Dynamic Interactions of Vapor Compression Heat Pumps,** *Energy. Cons. Mgmt.*, 24, pp. 49-66 (1984).

[9] McLinden, M.O., Gallagher, J.S., Weber, L.A., Morrison, G., Ward, D., Goodwin, A.R.H., Moldover, M.R., Schimdt, J.W., **Measurements and Formulation of the Thermodynamic Properties of Refrigerants 134a (1,1,1,2-Tetrafluoroethano) and 123 (1,1-Diclhoro-2,2,2-Trifluorethane),** *ASHRAE Transactions*, vol. 95, pt.2, 1989.

[10] Parise, J.A.R., **Simulation of Vapor-Compression Heat Pumps**, *Simulation*, 46:2, 71-76 (1986).

[11] Shankland, I.R., Basu, R.S., Wilson, D.P., **Thermal Conductivity and Viscosity of a new Stratospherically Safe Refrigerant - 1,1,1,2-Tetrafluoroethano (R134a),** *GENETRON Products Laboratory*,(1990).

[12] Wilson, D.P., Basu, R.S., **Thermodynamic Properties of a new Stratospherically Safe Working Fluid - Refrigerant 134A,** *ASHRAE Transactions*, vol. 94, pt.2, 1988.

[13] Yasuda, H., Touber S., Machielsen, C.H.M., **Simulation Model of a Vapor Compression Refrigeration System,** *ASHRAE Transactions*, Vol. 89, part 2A, no. 2787, pp. 408-425, 1983.